Paper ID EH024: Modeling and Experimental Verification of Geometry Effects on Piezoelectric Energy Harvesters

T. Reissman[1], J. M. Dietl[1], E. Garcia[1]
[1] Sibley School of Mechanical and Aerospace Engineering,
Cornell University, Ithaca, NY, 14853, U.S.A.

Abstract — Energy harvesters have gained much attention as renewable energy source applications within wireless sensor technology. Focus has been directed mostly in two realms, maximizing energy output and efficient conversion via energy management circuitry. More analysis is still needed though on the fundamentals of operation in order to optimize for the size and amount of piezoelectric material needed for energy harvester applications. This work extends on the modeling of piezoelectric cantilevers by adding in the geometry of variable cross-sections, exploring standard rectangular designs and configurations with tapers and curvatures. By changing the geometry, a change in the beam strain profile is induced and thus a change in the voltage output. Experimental results are included to show actual performance outputs of each of the designs.

Keywords: Energy harvesting, piezoelectrics, modeling, variable geometry, variable cross-sectional area

INTRODUCTION

Piezoelectrics have found themselves to be useful in applications such as actuators, sensors, and electric power generators, or energy harvesters. Earlier work performed by Hagood et al. [1] provided a physical model of piezoelectric cantilevers actuators using Hamilton's principle. A decade later, increasing interest in the functionality of piezoelectrics as generators transducing electric energy from vibration energy began. Goldfarb and Jones [2] showed that the efficiency of piezoelectic generators was approximately 10%. Goldfarb claimed that much of this inefficency in electric power generation was due to the electrical energy remaining stored in the piezoelectric and not transferred. Further sudies by researchers Roundy and Wright [3] and Sodano et al. [4] modeled standard rectangular beams with a cantilever configuration, using methods such as equivalent electrical models and the energy method.

The novelty of the work in this paper lies in the addition of variable cross-sections to the piezoelectric model for the purpose of evaluating geometric effects on the performace of piezoelectric energy harvesters. The cantilever beam shapes investigated include the standard rectangular design, a tapered design, and a parabolic design. The Rayleigh-Ritz method is used for calculating the performance output of these prismatic and non-prismatic beams.

Fig. 1 Rectangular, Tapered, & Parabolic (with tip mass) Beam Designs (observed from left to right)

By investigating these geometries, a reduction in the amount of piezoelectric material is calculable for applications in which lightweight energy harvesters are required. One such application is the Hybrid-Insects MEMS program within DARPA. In this program, cybernetic flying insects are created, which have sub-one gram systems onboard consisting of sensors, actuators, and energy harvesting devices. In designing these types of systems, the mass is a critical factor in order to still enable flight. Thus, researchers such as Reissman et al. [5] have been developing systems with high specific power, lightweight energy harvesters for such constraints.

ANALYTICAL MODEL

The piezoelectric beams are governed by Hamiltonian mechanics combined with piezoelectric constitutive laws. As in Hagood et al. [1] and Sodano et al. [4] the beam's displacement coordinate can be replaced with Rayleigh-Ritz modal functions:

$$y(x,t) = \boldsymbol{\varphi}(x)^T \boldsymbol{q}(t) \tag{1}$$

the integral of action for the beam motion becomes:

$$\begin{aligned} L = \int_{t_1}^{t_2} \Bigg(& \delta \Bigg\{ \frac{1}{2}\dot{\boldsymbol{q}}^T(t)\left[\int_0^L \rho_l w(x)\boldsymbol{\varphi}(x)\boldsymbol{\varphi}(x)^T\,dx\right]\dot{\boldsymbol{q}}(t) \\ & + \frac{1}{2}\dot{\boldsymbol{q}}^T(t)\left[M_0\boldsymbol{\varphi}(L)\boldsymbol{\varphi}^T(L)\right]\dot{\boldsymbol{q}}(t) \\ & - \frac{1}{2}\boldsymbol{q}^T(t)\left[\frac{I_{0w}}{s^E}\int_0^L w(x)\boldsymbol{\varphi}(x)''\,\boldsymbol{\varphi}(x)^{T''}\,dx\right]\boldsymbol{q}(t) \\ & - \dot{\lambda}\left[\frac{d}{s^E}(t_s+t_p)\int_0^L w(x)\boldsymbol{\varphi}(x)^{T''}\,dx\right]\boldsymbol{q}(t)\Bigg\} \\ & + \left[\int_0^L \rho_l w(x)\boldsymbol{\varphi}(x)^T\,dx + M_0\boldsymbol{\varphi}^T(L)\right]a(t)\delta q(t) \\ & + \frac{1}{R}\dot{\lambda}\delta\lambda\Bigg)dt \end{aligned} \tag{2}$$

where ρ_l is the mass per unit width and length (kg/m^2), $w(x)$ is the width of the beam at a distance x from the cantilevered root, M_0 is the mass of a tip mass, I_{0w} is the