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Modeling of nanofabricated paddle bridges for resonant mass sensing

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The modeling of nanopaddle bridges is studied in this article by proposing a lumped-parameter mathematical model which enables structural characterization in the resonant domain. The distributed compliance and inertia of all three segments composing a paddle bridge are taken into consideration in order to determine the equivalent lumped-parameter stiffness and inertia fractions, and further on the bending and torsion resonant frequencies. The approximate model produces results which are confirmed by finite element analysis and experimental measurements. The model is subsequently utilized to quantify the amount of mass which attaches to the bridge by predicting the modified resonant frequencies in either bending or torsion. © *2006 American Institute of Physics.* [DOI: 10.1063/1.2221560]

I. INTRODUCTION

Based on lumped-parameter modeling, this article studies the design and performance of paddle bridges that have dimensions in the nanometer range and are mainly utilized as sensors in mass deposition and as flexible components in micro-/nanoelectronics, such as switches. Constructively, the paddle bridge (whose dimensions are given in the sketch of Fig. 1) is a symmetric structure consisting of a middle portion, the paddle, and two identical end segments which also connect the freestanding structure to the substrate. Typically, this compliant structure has constant thickness (t) and the paddle segment is wider than the flexible end connectors ($w_2 > w_1$). Out-of-the-plane bending (about the z axis) and torsion (about the x axis)—as indicated in Fig. 1—are the principal modes of operation for bridges and the flexibility in both modes is mostly and desirably provided by the end segments. The wider paddle segment usually serves to provide the necessary surface for either external substance attachment detection or bending/torsion and actuation/sensing, and is the segment credited with inertia properties. A mathematical model of the nanopaddle oscillator was first provided by Dowell and Tang[1] who derived the out-of-the-plane (bending) resonant frequency of this device by using Lagrange's equations and the assumption that the end beams furnish the compliance of the entire device.

The paddle bridge has been first utilized in the 1850s (Ref. 2) as an electrometer by Thomson (Lord Kelvin) to measure electric potentials. The original device, called quadrant electrometer, consisted of a doubly clamped metallic wire with an aluminum paddle attached at its midpoint; the rotation angle of the paddle was proportional to the electric potential to be detected. Another report on a variety of paddle bridge oscillator, which is slightly different from the one pictured in Fig. 1, was given by Kleiman *et al.*,[3] who designed and characterized single-crystal silicon structures having dimensions in the millimeter range and that were used as torsion resonators for measuring the properties of thin liquid-crystal films. A similar design, also with dimensions in the millimeter range, was analyzed by Haiberger *et al.*,[4] who utilized the so-called double-paddle oscillator (DPO) to detect the dynamical gravitational field. Cleland and Rourke,[5,6] introduced the nanometer-scale mechanical electrometer to detect electric charge in the presence of a magnetic field. Studied in their works was the electric-magnetic interaction, which created torsional vibrations of a paddle bridge nanodevice, whose vibratory motion further induced an electromotive force that is proportional to the electric charge. In micro-/nanoelectronics, the paddle bridge has been utilized in applications such as signal routing or phase shifting, actual implementations consisting of digitized/tunable filters or capacitors—(Brown[7]).

Another application of the paddle bridge is the detection