Circular-Hinge Line Element for Finite Element Analysis of Compliant Mechanisms

Nicolae Lobontiu*
e-mail: nl73@cornell.edu

Ephrahim Garcia
Sibley School of Mechanical and Aerospace Engineering,
Cornell University,
258 Upson Hall,
Ithaca, NY 14853

A three-node six degree-of-freedom per-node line element that is sensitive to axial, bending, and torsional loading is introduced to model single-axis right circular hinges of constant width that are utilized in compliant mechanisms. The Timoshenko model is applied for bending because this particular configuration is virtually short, and provisions are taken that the element is shear-locking free. The Saint Venant theory, which includes warping, is utilized to model torsion of the variable rectangular cross-section circular hinge. The principle of minimum total potential energy is employed to formulate the elemental stiffness and mass matrices, as well as the elemental nodal vector. Static force deflection and modal simulation that are performed based on this finite element model produce results that are in agreement with simulation by commercially available finite element software. The three-node line element is also compared to an analytical model in terms of stiffness and the results are again concurring. [DOI: 10.1115/1.1825046]

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Introduction

The flexure hinges are structural components that are utilized in compliant mechanisms to connect rigid links and produce limited relative rotation through elastic bending under external loading and/or actuation. Often, the flexure hinges are constructed monolithically with the rest of the compliant mechanism, which adds to the intrinsic virtues of these members and make them attractive over classical rotation joints. Advantages presented by flexure hinges include ease of fabrication, no assembly (in the case of monolithic flexures), no friction losses, no (or very low) hysteresis and no maintenance. There are also a few inevitable limitations in using flexure hinges, such as limited levels of rotation, nonpure rotation (because the flexure hinge is also sensitive to the so-called ''parasitic effects,'' such as axial and shearing loading and which accompany the pure bending) and thermal sensitivity.

The flexure hinges are employed in a wide range of applications in the automobile, aviation, medical, computer, or fiber optic industries, and a few examples in these areas include precision positioning mechanisms, suspension devices, actuators, sensors, flexible couplings, catheters, orthotic/biopsy devices, print heads, keyboards, microscopes, or optical heads. The flexure hinges lend themselves to implementation into compact microscale devices or microelectromechanical systems (MEMS) such as optical switches, microactuators, microsensors, microgyroscopes, microfluidic devices, microcantilevers for microscopes or tilt mirrors. The example of Fig. 1(*a*) shows the simplified construction of a MEMS microswitch device, whereby a flexure hinge elastically connects a fixed anchor to a movable plate that can be attracted through electrostatic forces by a fixed plate such that the on/off switching function can be produced in an electric circuit.

It can be seen in Fig. 1(*a*) that the relative rotation between the adjacent rigid links is achieved through bending the monolithic hinge, which is called flexure hinge in this case. Figure 1(*a*) also suggests that the bending axis (also called sensitive axis), which is passing through the symmetry center of the flexure, can be set in two different positions, depending on the particular design and functional role of one flexure hinge.

Another example, the one of a torsional MEMS device, is sketched in Fig. 1(*b*). Lateral actuation of the middle plate will produce torsion of the aligned hinges, so that the entire device will rotate about the longitudinal axes of the two hinges, which will coincide with the torsion-sensitive axis. The hinge designs shown in Figs. 1(*a*) and 1(*b*) have one sensitivity axis, such that the hinge stiffness about that particular axis is minimum and the motion-through-deformation is enabled about that axis.

The analytic approach to flexure hinges was introduced in the 1960's in a paper by Paros and Weisbord [1], which treated the symmetric single-axis circular flexure hinges. The compliances (or spring rates) of such flexures were given analytically, both in exact and simplified forms, for in-plane and out-of-the-plane sensitivity to bending and axial loading. Smith et al. [2] extrapolated the procedure of Paros and Weisbord [1] to give approximate compliance equations for symmetric single-axis elliptic flexure hinges. Lobontiu et al. [3] used the Castigliano's displacement theorem to derive the exact in-plane and out-of-the-plane compliance equations that define symmetric single-axis corner-filleted flexure hinges. A similar procedure was utilized by Lobontiu et al. [4] to formulate the exact closed-form compliance equations for the class of multiple-axis circular-section flexure hinges. A more vast collection of flexure designs from the single-, two-, and multiple-axis categories, together with the corresponding closed-form compliance equations can be found in Lobontiu [5].

Saxena and Ananthasuresh [6] proposed a synthesis method for compliant mechanisms by studying geometrically nonlinear (large displacements) finite elements. A sensitivity analysis was also performed on several examples that are illustrative of the method. Xu and Ananthasuresh [7], more recently, proposed a freeform shape optimization algorithm dedicated to skeletal structures. Remarkable work in the domain of compliant mechanisms that undergo large deformations include the work of Howell [8] and Howell and Midha [9,10]. Similarly, Kimball and Tsai [11] analyzed the flexural beams in compliant mechanisms by means of the pseudo rigid body model. Bert et al. [12] analyzed the nonlinear response of torsional couplings that include flexible links. Carricato, Parenti-Castelli, and Duffy [13] presented the inverse static analy-

*To whom correspondence should be addressed.

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