Two Microcantilever Designs: Lumped-Parameter Model for Static and Modal Analysis

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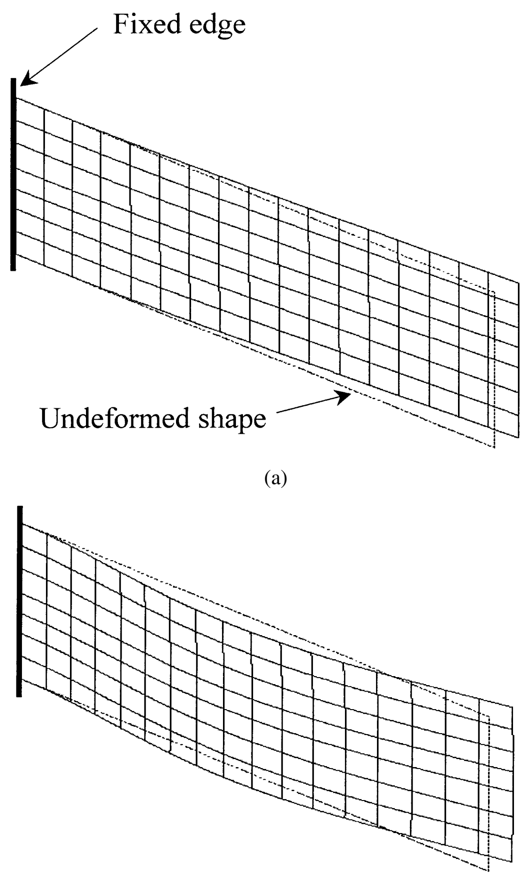
***Abstract*—This paper develops a lumped-parameter analytical model to enable static load-deformation and modal analysis for two microcantilever designs that can be utilized in mass detection and AFM applications. The circularly filleted microcantilever is currently utilized in several applications, whereas the elliptically filleted configuration is novel. Closed-form compliance/stiffness equations are derived for both microcantilevers in lumped form to permit design in the static regime for both long and short members. Lumped effective inertia fractions are also formulated for the two designs and used in conjunction with the lumped stiffness fractions to quantify the modal response. It is thus possible to tune the static and modal responses of these specific microcantilever designs by geometry alterations. The analytical equations are confirmed through limit calculations, finite element simulation, and experimental results available in the literature. [1011]**

***Index Terms*—Atomic force microscopy (AFM), mass sensing, microcantilever, lumped-parameter model, compliance/stiffness, modal frequency.**

I. Introduction

THE microcantilevers are utilized in several atomic force microscopy (AFM) applications such as nanoindentation, optical position detection down to Angstrom/sub-Angstrom resolution, as well as in a host of sensing/actuation devices including various piezoresistive, piezoelectric, or thermal designs. The application range of microcantilevers also includes microsystems for imaging of surface topology, measurement/probing of material elastic and strength properties, writing/reading on surfaces, high-aspect ratio metrology, metallography, chemical/electrochemical characterization, microlubrication/tribology, corrosion processes, cellular engineering or grain growth and surface adhesion phenomena. The microcantilevers can be utilized either in the static/quasi-static regime—in order to measure deflections, or the oscillating mode—when the modal frequencies are monitored. Some of the many and diverse applications of microcantilevers will briefly be discussed next.

Ilic *et al.* [1] presented a circularly filleted microcantilever design that can be used for detecting very small amounts of material through monitoring alterations in the first resonant frequency. In a similar manner, Raiteri *et al.* [2] gave a review of the utilization of microcantilevers as biosensors and discussed various methods of detecting the additional mass. Lange *et al.* [3] presented applications such as beam resonators, gas sensors, and force sensors for parallel scanning AFM. In AFM applications, the microcantilevers are generally designed to either indent (write) a pattern at microscopic scale with nanoprecision or to detect (read) an already existing topology. A brief sample of examples where the microcantilevers are implemented in write/read AFM applications include the works of Morita *et al.* [4], Garcia and Perez [5], Vie *et al.* [6], Shao *et al.* [7], van de Water and Molenaar [8], Snow *et al.* [9], Chui *et al.* [10], Ried *et al.* [11], King *et al.* [12] or Peterson *et al.* [13], to cite just a few.



(a)

(b)

Fig. 1. Bending modes in a constant rectangular cross-section fixed-free beam. (a) Tip mode, (b) Midspan mode.

Harley *et al.* [14] and Harley and Kenny [15] proposed guidelines for the mechanical design of resonant force transducers, and concluded that narrow microcantilevers are optimal for resonant sensing, especially when they are oriented axially and have high axial spring constants. Kageshima *et al.* [16] proposed two types of cantilever-based force sensors that were designed

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