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Stiffness characterization of corner-filleted flexure hinges

Nicolae Lobontiu[a] and Ephrahim Garcia
Cornell University, Sibley School of Mechanical and Aerospace Engineering, Ithaca, New York 14850

Mihail Hardau and Nicolae Bal
Technical University of Cluj-Napoca, Romania

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The paper formulates the closed-form stiffness equations that can be used to characterize the static, modal, and dynamic behavior of single-axis corner-filleted flexure hinges, which are incorporated into macro/microscale monolithic mechanisms. The derivation is based on Castiliagno's first theorem and the resulting stiffness equations reflect sensitivity to direct- and cross-bending, axial loading, and torsion. Compared to previous analytical work, which assessed the stiffness of flexures by means of compliances, this paper directly gives the stiffness factors that completely define the elastic response of corner-filleted flexure hinges. The method is cost-effective as it requires considerably less calculation steps, compared to either finite element simulation or experimental characterization. Limit calculations demonstrate that the known engineering equations for a constant cross-section flexure are retrieved from those of a corner-filleted flexure hinge when the fillet radius becomes zero. The analytical model results are compared to experimental and finite element data and the errors are less than 8%. Further numerical simulation based on the analytical model highlights the influence of the geometric parameters on the stiffness properties of a corner-filleted flexure hinge. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1806999]

I. INTRODUCTION

The flexure hinges are utilized in many engineering applications, both macro- and micro/nano-scale (MEMS/NEMS), as flexible connectors that provide relative motion between adjacent rigid links by means of elastic deformation that is generally being produced in the active domain through either bending or torsion. Examples of flexure hinge implementations include precision positioning devices, scientific/laboratory mechanisms, actuators, antifriction bearings, biomedical devices, optical mirrors, resonators, accelerometers, gyroscopes, and microcantilevers.

This paper derives the closed-form equations for the stiffness factors of single-axis corner-filleted flexure hinges. The published literature dedicated to the analytical treatment of flexure hinges has developed thus far models based on compliance equations, either exact or approximate. This approach has been enabled by calculation procedures that are particularly suited to this type of analysis, such as Castigliano's second (displacement) theorem, the reciprocity theorem or the principle of virtual work. Paros and Weisbord[1] studied the circular flexure hinges and derived the spring rates (compliances), both exact and approximate (for engineering use), as deformation-load equations that depend exclusively on geometry and material properties. Weinstein,[2] in a somewhat similar approach, provided the spring rates and the corresponding design diagrams for one-, two- and three-strip flexure-based pivot bearings. Based on extrapolating the approach of Paros and Weisbord,[1] Smith and Badami[3] developed approximate equations for the spring rates defining the elliptic flexure hinges, and verified the model accuracy by both finite element analysis and experimental investigation. Xu and King[4] employed commercially-available finite element software to analyze and compare right circular, corner-filleted and elliptic flexure hinges, and their conclusion was that the corner-filleted configuration performs the best in terms of motion precision. Lobontiu, Paine, Garcia, and Goldfarb[5] applied Castigliano's second theorem in order to derive exact closed-form compliance equations for corner-filleted flexure hinges. A similar approach was followed by Lobontiu and Paine[6] in introducing the multiple-axis circular-cross section (revolute symmetry) corner-filleted flexure hinges, and by Lobontiu and Garcia[7] in proposing the compliance-based formulation for two-axis with axially-collocated notches flexure hinges. Several other single-, two- and multiple-axis flexure configurations can be found in Lobontiu.[8] Hsiao and Lin[9] proposed a novel flexure design and the compliance-based analytical modeling for a serial chain comprising two right circular flexures and a three-quarter circle configuration. Wu and Zhou,[10] more recently, revisited the spring rate equations originally produced by Paros and Weisbord[1] for circular flexure hinges, and provided an alternative formulation. Tseytlin[11] also focused on the compliances of circular flexure hinges, by following the method of inverse conformal mapping and by including the shearing effects into the formulation.

Finite element procedures have also been developed more recently as an alternative to the analytical modeling and the finite element analysis by commercially-available codes. Lobontiu[8] gave a generic formulation for single-, two- and multiple-axis flexure hinges as three node line elements,

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[a] Author to whom correspondence should be addressed; electronic mail: n173@cornell.edu