Available online at www.sciencedirect.com
SCIENCE DIRECT®

Computers and Structures 81 (2003) 2797–2810

Computers & Structures

www.elsevier.com/locate/compstruc

Analytical model of displacement amplification and stiffness optimization for a class of flexure-based compliant mechanisms

Nicolae Lobontiu *, Ephrahim Garcia

Sibley School of Mechanical and Aerospace Engineering, Center for Intelligent Machine Systems, Cornell University, 258 and 224 Upson Hall, Ithaca, NY 14853, USA

Received 7 February 2003; accepted 24 July 2003

Abstract

The paper formulates an analytical method for displacement and stiffness calculations of planar compliant mechanisms with single-axis flexure hinges. The procedure is based on the strain energy and Castigliano's displacement theorem and produces closed-form equations that incorporate the compliances characterizing any analytically-defined hinge, together with the other geometric and material properties of the compliant mechanism. Displacement amplification, input stiffness and output stiffness calculations can simply be performed for any serial compliant mechanism. The class of amplifying compliant mechanisms that contain symmetric corner-filleted or circular flexure hinges is specifically addressed here. A parametric study of the mechanism performance is performed, based on the mathematical model, and an optimization procedure is proposed, based on Lagrange's multipliers and Kuhn–Tucker conditions, which identifies the design vector that maximizes the performance of these flexure-based compliant mechanisms. Independent finite element simulation confirms the analytical model predictions.
© 2003 Elsevier Ltd. All rights reserved.

Keywords: Flexure hinge; Compliant mechanism; MEMS; Amplification; Stiffness

1. Introduction

Flexure-based compliant mechanisms are increasingly utilized in both macro-scale and micro-scale (MEMS) applications where precision of motion, reliability, ease of fabrication, compactness and feasibility (especially) in micro-environments are needed. In MEMS, for instance, realizing the necessary mobility between adjacent members is almost entirely achieved through flexible connectors, such as flexure hinges. Examples of implementation of these mechanisms are numerous and they can be found in the automotive, aerospace, telecommunications, medical, optics and computer industries.

This paper formulates a mathematical model that can be utilized in the design and optimization of planar compliant mechanisms that gain their mobility by means of flexure hinges. The analytical model studies the static/quasi-static response of these mechanisms by incorporating the compliances that define the elastic behavior of various single-axis flexure configurations into generic load-deformation/displacement closed-form equations. By means of these equations it is possible to simply select a design, analyze it and optimize its behavior in terms of performance criteria such as displacement (geometric) amplification and/or input/output stiffness. The proposed calculation tool is both precise (being based on exact closed-form equations) and simple to implement, as an alternative to the often-time costly

* Corresponding author. Tel.: +1-607-255-4379; fax: +1-607-255-1222.

E-mail address: nl73@cornell.edu (N. Lobontiu).

0045-7949/$ - see front matter © 2003 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compstruc.2003.07.003